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UNSTEADY, CONJUGATED, FORCED CONVECTION HEAT TRANSFER IN A PARALLEL PLATE DUCT

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Abstract—This work deals with the unsteady, conjugated heat transfer problem of a parallel plate duct with inlet fluid temperature varying periodically in time. Previous approaches have utilized either the usual, standard, quasi-steady approach or have employed a slug flow velocity profile. Here, the actual quadratic velocity profile, characteristic of fully developed duct flow, is used in an improved quasi-steady approach, previously introduced by the senior author, which takes into approximate account the effects of both thermal history and finite thermal capacity of the fluid. Response functions for the duct wall temperature and fluid bulk mean temperature are developed using this approach and are compared to a benchmark finite-difference solution as well as to the solution by the usual, simple quasi-steady approach.

NOMENCLATURE

- a — ratio of thermal energy storage capacity of fluid to that of the solid, $\rho c_p R/\rho_w c_{pw} b$
- b — wall thickness of duct
- b^* — $\rho_w c_{pw} b R\omega/k$
- C^* — defined in equation (23)
- c_p, c_{pw} — specific heat of fluid and wall, respectively
- G — kernel in equation (4)
- G_n — constants in equation (5)
- h — surface coefficient of heat transfer
- j — index related to the number of terms in s retained in equation (13)
- k — thermal conductivity of the fluid
- n — index
- Nu — Nusselt number, hR/k
- $P(s), Q_1(s), Q_2(s)$ — polynomials in s given by equation (17)
- q_w — surface heat flux
- Q_w — nondimensional surface heat flux, $-\chi^{1/3} q_w R/3(0.538)\Gamma(\frac{2}{3})k\Delta T_0$
- r_0 — defined in equation (15)
- R — half height of duct
- s — Laplace transform parameter
- sqs — subscript meaning standard quasi-steady method
- t — time
- u_m — mass average velocity
- x, y — space coordinates along and perpendicular to duct wall, respectively.

Greek symbols

- α — thermal diffusivity of the fluid, $k/\rho c_p$
- β — dummy variable of integration
- Γ — complete gamma function
- ΔT_0 — amplitude of sinusoidal inlet temperature variation
- θ — temperature excess
- θ_w, θ_B — wall and fluid bulk mean temperature excess, respectively
- ρ, ρ_w — mass density of fluid and wall, respectively
- σ — defined by equation (10)
- Σ_j — defined by equation (14)
- χ — nondimensional axial coordinate, $\alpha x/R^2 u_m$
- τ — $t - x/u_m$
- λ_n — eigenvalues needed in equation (5)
- ϕ_n — defined by equation (7).

INTRODUCTION

IN A PARALLEL flow regenerative type heat exchanger, the unsteady temperature distribution within the wall material is mutually coupled to that within the moving fluid and, hence, causes a conjugated heat transfer situation in which the problem is to predict the wall temperature and the bulk mean temperature of the fluid as a function of position and time. The simplest, most expedient approach to this problem, and the one most often used, is the standard, simple, quasi-steady approach which employs a constant, in space and time, surface coefficient of heat transfer h. Willmott and Burns [1] utilize this approach, in conjunction with numerical methods, to solve the classical regenerator equations. Kardas [2], and, more recently, Spiga and Spiga [3] have also used the standard quasi-steady method, along with the Laplace transformation, to solve regenerator type problems. For the case where the velocity profile is idealized as being a slug flow, a number of exact analytical solutions have been developed. Chase *et al.* [4] used Laplace transforms to arrive at a solution when there is a step change in the duct inlet fluid temperature while Sparrow and DeFarias [5] used the method of complex temperature to predict the wall and fluid bulk mean temperature, in the periodic unsteady state, due to a sinusoidal inlet temperature variation. Sucec [6] developed an exact solution that includes both the periodic unsteady state and the transient unsteadiness that leads up to this state. Acker and Fourcher [7] include the effect of transverse conduction in the wall of the duct considered in ref. [5].

Thus, previous approaches have either used the standard, simple quasi-steady technique or else have dealt with a slug velocity profile. The work presented herein will utilize the actual quadratic velocity profile, characteristic of fully developed duct flow, in an improved quasi-steady approach, introduced in Sucec [8], which takes into approximate account both the effects of thermal history and of finite thermal capacity of the fluid, effects not accounted for in the standard quasi-steady approach. Wall temperature and fluid bulk mean temperature are found as a function of distance down the duct and time, in the periodic unsteady state, for a sinusoidal inlet temperature variation when the duct walls are adiabatic on their outside surfaces and communicate thermally with the fluid across their inside surfaces. These predictions are compared with the true response functions as found by a finite-difference solution of the governing partial differential equations to establish the domain of validity of the improved quasi-steady approach in this problem. Also presented, for comparison purposes, is the standard, simple quasi-steady prediction.

ANALYSIS

To be considered is a parallel plate duct of half height R with walls of thickness b which are perfectly insulated on their outer surfaces as a result, perhaps, of thermal and geometric symmetries. Fluid, whose inlet temperature is $\Delta T_0 \sin \omega t$, flows through the duct in a steady, laminar, constant property, fully developed hydrodynamical fashion. Axial conduction is taken to be negligible in both the fluid and the duct wall, wall thermophysical properties are constant, and transverse temperature gradients in the wall are small. Discussion of the validity of these assumptions is given in Sucec [6].

Energy balances on the fluid and on the wall lead to the following exact mathematical problem description

$$\frac{\partial \theta}{\partial t} + 3u_m \left[\frac{y}{R} - \frac{y^2}{2R^2}\right]\frac{\partial \theta}{\partial x} = \alpha \frac{\partial^2 \theta}{\partial y^2}. \quad (1)$$

At

$$y = 0, \quad t > 0, \quad x > 0, \quad \rho_w c_{pw} b \frac{\partial \theta}{\partial t} - k\frac{\partial \theta}{\partial y} = 0. \quad (2)$$

At

$$y = R, \quad t > 0, \quad x > 0, \quad \frac{\partial \theta}{\partial y} = 0, \quad x = 0, \quad t > 0,$$

$$0 \leqslant y \leqslant R, \quad \theta = \Delta T_0 \sin \omega t. \quad (3)$$

No initial condition is needed since the solution is being sought in the periodic state.

Improved quasi-steady solution

In the approximate solution to equations (1)–(3), by the improved quasi-steady approach presented in ref. [8], the expression for the instantaneous surface heat flux q_w is given as

$$q_w = \int_0^\chi G[\chi-\beta]\frac{\partial \theta_w}{\partial \beta}(\beta,\tau)\,d\beta, \quad (4)$$

for the second time domain, $t \geqslant x/u_m$ or $\tau \geq 0$, where the kernel of equation (4), $G(\chi)$, is related to the solution of the case of steady-state heat transfer in a duct with isothermal walls. Details are available in ref. [8] where a related problem was solved using, for analytical convenience, a linear velocity profile in the thermal entrance region of a duct which led to a fairly simple expression for $G(\chi)$. In this work, however, the considerably more complex kernel appropriate to the actual quadratic velocity profile, shown in equation (1), is to be used and is available in Kays [9] as

$$G(\chi) = \frac{k}{R}\sum_{n=0}^{\infty} G_n \exp\left[-\lambda_n^2\chi/8\right]. \quad (5)$$

Inserting equation (5) into equation (4), using the resultant expression for q_w to replace $-k(\partial\theta/\partial y)_{y=0}$ in the energy balance on the wall, equation (2), and changing variables to those dictated by the improved quasi-steady method of ref. [8] yields the governing equation for the wall temperature distribution as

$$\frac{\partial \theta_w}{\partial(\omega\tau)} + \frac{1}{b^*}\int_0^\chi \left\{\sum_{n=0}^{\infty} G_n \exp\left[-\phi_n(\chi-\beta)\right]\right\} \times \frac{\partial \theta_w}{\partial \beta}\,d\beta = 0, \quad (6)$$

$$\phi_n = \lambda_n^2/8. \quad (7)$$

Since a convolution integral appears in equation (6), the solution was attempted by use of a Laplace transformation with respect to χ which, after application of the second condition in equation (3), gives

$$\frac{d\bar{\theta}_w}{d(\omega\tau)} + \frac{1}{b^*}\sum_{n=0}^{\infty}\frac{G_n}{s+\phi_n}\left[s\bar{\theta}_w - \Delta T_0 \sin \omega\tau\right] = 0. \quad (8)$$

Solving equation (8) and retaining only the periodic unsteady-state portion of the solution gives

$$\frac{\bar{\theta}_w}{\Delta T_0} = \frac{\sigma^2 s \sin \omega\tau}{1+\sigma^2 s^2} - \frac{\sigma \cos \omega\tau}{1+\sigma^2 s^2}, \quad (9)$$

$$\sigma = \frac{1}{b^*}\sum_{n=0}^{\infty}\frac{G_n}{s+\phi_n}. \quad (10)$$

The main difficulty in finding the inverse Laplace transformation of equation (9) is due to the fact that σ is an infinite series in which every term contains the transform parameter s. Initially, it was thought that equation (9) could be cast into a more readily handled form by a suitable truncation of equation (6) at large χ in the physical (τ, χ) plane. However, it is noticed that the accounting for previous history leads to an exponential term in equation (6) whose argument is small, not large, regardless of the size of χ since β takes on values from 0 to χ. Hence, a simplification was sought in the transform plane instead. Expanding equation (10) results in

$$\sigma = \frac{1}{b^*}\left[\frac{G_0}{s+\phi_0}+\frac{G_1}{s+\phi_1}+\frac{G_2}{s+\phi_2}+,\ldots, +\frac{G_n}{s+\phi_n}\cdots\right]. \quad (11)$$

Now, since large χ corresponds to small values of s and since $\phi_0 < \phi_1 < \phi_2 < \cdots \phi_n < \cdots$, then, for large enough χ, one can say that at some j

$$s+\phi_{j+1} \approx \phi_{j+1}. \quad (12)$$

With equation (12), σ can be re-written as

$$\sigma = \sigma_j = \frac{1}{b^*}\left[\sum_{n=0}^{j}\frac{G_n}{s+\phi_n}+\Sigma_j\right], \quad (13)$$

$$\Sigma_j = \sum_{n=j+1}^{\infty}\frac{G_n}{\phi_n}. \quad (14)$$

Using equation (13) for σ, the numerator and denominator of equation (9) become finite polynomials in s which can be inverted. The procedure is to first take $j = 0$, then $j = 1, 2$, and so on, invert equation (9) and compare the $\theta_w/\Delta T_0$ results at the value of χ of interest and when the results are essentially the same at two successive values of j, the required size of j is established.

For $j = 0$, using equations (13) and (14) in equation (9) and defining

$$r_0 = G_0/\Sigma_0, \quad (15)$$

yields

$$\frac{\bar{\theta}_w}{\Delta T_0} = \frac{Q_1(s)\sin\omega\tau - Q_2(s)\cos\omega\tau}{P(s)}, \quad (16)$$

$$Q_1(s) = s^3+2s^2(r_0+\phi_0)+s(r_0+\phi_0)^2,$$

$$Q_2(s) = \frac{b^*}{\Sigma_0}[s^2+s(r_0+2\phi_0)+\phi_0(\phi_0+r_0)], \quad (17)$$

$$P(s) = s^4+2s^3(r_0+\phi_0)+s^2(r_0^2+2r_0\phi_0+\phi_0^2+b^{*2}/\Sigma_0^2)+(b^{*2}/\Sigma_0^2)\phi_0(2s+\phi_0).$$

In order to invert equation (16), the roots of the polynomial $P(s)$ must be found and this was accomplished by the use of a standard library program, for root finding, at the Computer Center of the University of Maine. For the values of b^* investigated, all the roots of $P(s)$ were complex, complex conjugate pairs whereas in the limits as $b^* \to 0$ and $b^* \to \infty$ the roots were found to be all real. From the definition of b^*, the limits $b^* \to \infty$ and $b^* \to 0$ correspond to infinite wall thermal capacity, or high frequency, and zero wall thermal capacity, or low frequency, respectively. On a physical, intuitive, basis therefore, $b^* \to \infty$ would lead to a wall temperature which is constant at its initial value and $b^* \to 0$ would give a wall temperature which is equal to the time varying fluid inlet temperature. Formal investigation of these two limits of b^* by way of equations (9) and (10) leads to the same conclusions. At these limits, the ratios of the polynomials of equation (17) become $Q_1(s)/P(s) = Q_2(s)/P(s) \to 0$ as $b^* \to \infty$ and $Q_1(s)/P(s) \to 0$, $Q_2(s)/P(s) \to 1/s$ as $b^* \to 0$.

With the roots obtained, the inversion of equation (9) follows with the aid of a table of transforms, Roberts and Kaufman [10], and gives the following wall temperature distribution for the $j = 0$ one-term solution

$$\frac{\theta_w}{\Delta T_0} = \sin\omega\tau\{e^{-a_1\chi}(b_1\cos c_1\chi+b_2\sin c_1\chi) +e^{-a_2\chi}(b_3\cos c_2\chi+b_4\sin c_2\chi)\} -\cos\omega\tau\{e^{-a_1\chi}(b_5\cos c_1\chi+b_6\sin c_1\chi) +e^{-a_2\chi}(b_7\cos c_2\chi+b_8\sin c_2\chi)\}. \quad (18)$$

In equation (18), the a_1, a_2, b_1, b_2, c_1, etc. depend upon the numerical value of b^* used in equation (17). More of the details as well as the expressions for $j = 1$ and 2 are available in ref. [11].

Since, as shown in Sucec [8], the improved quasi-steady technique is an exact solution for slug flow, a check on the overall integrity of the solution procedure of the present work was made by comparing it to the exact analytical solution to the same problem for slug flow given by Sparrow and DeFarias [5]. Thus, using the coefficients and eigenvalues, $G_n = 2.0$, $\lambda_n^2/8 = \Pi^2(n+\frac{1}{2})^2$, appropriate to slug flow, solutions were obtained using $j = 0, 1$, and 2. Comparison of these results to those of ref. [5] for $b^* = 1$, 2, and 10, indicated complete agreement for $j \leqslant 2$ for values of the nondimensional axial coordinate $\chi \geqslant 0.10$. Thus, retention of three terms containing s in equation (13) which corresponds to the use of $j = 2$, allows one to make calculations as close to the duct inlet as $\chi = 0.10$. Also, comparison of solutions for θ_w, for $j = 0$ and 1, to that for $j = 2$ indicates that $j = 1$ suffices for $\chi \geqslant 0.20$ and just a single term in s, $j = 0$, is sufficient for $\chi \geqslant 1.0$.

Next, the solution to the problem by the improved quasi-steady method using the actual quadratic velocity profile will be outlined. In this case, the values of G_n and λ_n needed in equation (5) are available in Kays [9]. These values were employed in equations (13) and (14) for $j = 0, 1$, and 2 once again and lead to expressions (17) and (18) for $j = 0$, and to lengthier, similar, ones, for $j = 1$ and 2, that are available in ref. [11].

Now that the solution function for θ_w is available as equation (18) for $j = 0$ and as similar expressions for $j = 1$ and 2, the surface heat flux and the fluid's bulk mean temperature can be found

$$q_w = -\frac{b^*k}{R}\frac{\partial\theta_w}{\partial(\omega\tau)}. \quad (19)$$

An energy balance on a control volume of fluid dx

long gives the fluid's bulk mean temperature as

$$\theta_B/\Delta T_0 = \sin \omega\tau + b^* \int_0^\chi \frac{\partial \theta_w}{\partial(\omega\tau)}\, d\beta. \qquad (20)$$

Details of the actual forms of Q_w and θ_B for $j = 0, 1$, and 2 are given in ref. [11].

Finite-difference solution

In order to assess the performance of the just discussed improved quasi-steady approach in this problem, the solution to equations (1)–(3) was found by application of the finite-difference method. The finite-difference equations and procedure used are the ones described in some detail in ref. [8]. The lattice spacings, $\Delta\chi$, Δy and Δt were refined to the point where the finite-difference solution became independent of their size. As a check, the finite-difference solution to the steady state, isothermal wall, thermal inlet length problem was found and compared satisfactorily with the results of the exact analytical solution given in ref. [9].

Standard quasi-steady approach

The solution to the problem by the usual, standard quasi-steady approach employing a constant value of the surface coefficient of heat transfer, h, was also found. An energy balance on the wall and on the fluid with a constant value of h leads to a set of two mutually coupled partial differential equations for $\theta_{W_{sqs}}$ and $\theta_{B_{sqs}}$. For the periodic unsteady state, these equations were solved in ref. [5] by the method of complex temperature and, as a check, were solved again by the present authors by the Laplace transform method. It might be worthwhile noting that the solution by this standard, 'simple', quasi-steady approach, though it is less work than the improved quasi-steady approach discussed earlier, is nowhere near as straightforward as the solution to an external, boundary-layer flow problem. This is due to the fact that in the external flow problems the temperature of the free stream to be used in Newton's cooling law is a known constant, or a known function of time, whereas in this duct flow θ_B is an unknown function of time. The results for the wall and bulk mean temperature are

$$\frac{\theta_{W_{sqs}}}{\Delta T_0} = e^{-b^{*2} Nu\chi/(b^{*2}+Nu^2)} \left\{ \frac{Nu^2}{b^{*2}+Nu^2} \times \left[\cos(C^*\chi) - \frac{b^*}{Nu}\sin(C^*\chi)\right] \sin\omega\tau - \frac{Nu\, b^*}{b^{*2}+Nu^2}\left[\cos(C^*\chi) + \frac{Nu}{b^*} \times \sin(C^*\chi)\right]\cos\omega\tau \right\}, \qquad (21)$$

$$\frac{\theta_{B_{sqs}}}{\Delta T_0} = e^{-b^{*2} Nu\chi/(b^{*2}+Nu^2)} \sin[\omega\tau - C^*\chi], \qquad (22)$$

$$C^* = b^* Nu^2 \chi/(b^{*2}+Nu^2). \qquad (23)$$

DISCUSSION OF RESULTS

The improved quasi-steady method response functions for the wall temperature, fluid bulk mean temperature and wall heat flux were generated for the quadratic velocity profile by using the G_n and λ_n, given in Kays [9], in equations (13) and (14) using values of $j = 0, 1$, and 2. Then the use of equations (13) and (14) in equation (9) followed by inversion of equation (9) for the three different values of j establishes the number of terms in s in equation (13) which must be retained for convergence. When three terms are used ($j = 2$), it is found that this large χ procedure actually is valid even for χ as low as 0.02 which is extremely close to the duct entrance. Similarly $j = 2$ is needed for $\chi = 0.10$ while two terms in s in equation (13) suffice, $j = 1$, for $0.20 \leqslant \chi \leqslant 0.50$. For values of $\chi > 0.50$, only a single term in s is required, $j = 0$, and the wall temperature response is of the form given by equation (18). Based upon the more extensive results of ref. [11], these conclusions seem to be valid for all values of b^*.

Shown in Figs. 1 and 2 as the encircled points are some representative improved quasi-steady predictions of wall temperature and fluid bulk mean temperature for values of b^*, namely, 1.0 and 2.0, suggested by the work of ref. [5]. The plotted points are all for $j = 2$ since these solutions were found anyway in order to study and establish the convergence of the solution procedure. Also shown in Figs. 1 and 2 as solid lines for $\theta_w/\Delta T_0$, and as dashed lines for $\theta_B/\Delta T_0$, are the finite-difference solutions to equations (1)–(3) for $a = 0.20$ and with one curve for $a = 0.50$. Finally, the usual, standard quasi-steady results, equations (21) and (22) with a Nusselt number $Nu = 1.885$, are depicted either as dashed-dot curves or by crosses, depending upon which gives the greater clarity of presentation. In Fig. 1, at $\chi = 0.02$, the standard quasi-steady results for the bulk mean temperature plot are so close to the improved quasi-steady results and to the finite-difference solution that they were left off in the interests of clarity. Also, in Fig. 2, these were not shown on some of the curves since the deviation was not as substantial as on the corresponding curves in Fig. 1. Because of the facts that the periodic unsteady state is being considered and that the fluid inlet temperature is sinusoidal in time, only the first half cycle of each response function is presented since the second half is a simple image of the first half.

An overview of Figs. 1 and 2 shows the qualitative trends that would be anticipated from a knowledge of the slug flow solutions of refs. [5, 6], namely, the decay of the amplitude of the response, and the phase lag, as χ increases. As is evident from Fig. 1, particularly at $\chi = 0.02$, and 0.10, and for $\chi = 0.10$ in Fig. 2, the improved quasi-steady results practically coincide with the finite-difference baseline solutions while the standard quasi-steady results are considerably in error, particularly for the wall temperature. It is also seen, from Fig. 1, that as χ increases, the standard, simple, quasi-steady results get closer to the finite-difference

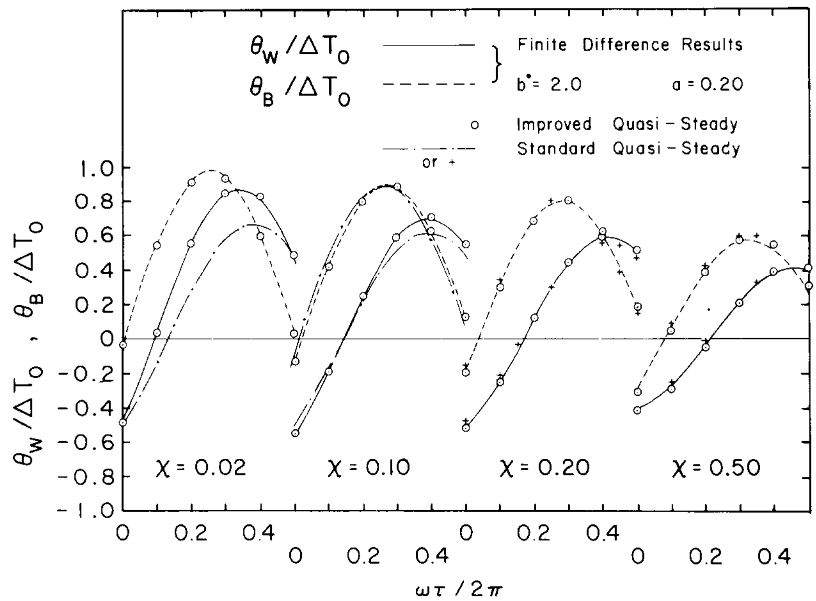

FIG. 1. Wall and bulk mean temperature excess distribution.

solutions and also to the improved quasi-steady solution. At the smaller values of χ, the standard quasi-steady solution is in error due to not accounting for surface temperature variation (thermal history) properly, nor for fluid thermal capacity, and also because of the effect of being in the thermal entrance region. On the other hand, the improved quasi-steady approach, being advanced here, uses an expression for the instantaneous heat flux, basically the second term of equation (6), which takes these effects into at least approximate account and this is reflected in the agreement between it and the finite-difference solution. At the larger χ values, such as $\chi = 0.50$ in Fig. 1, the standard quasi-steady approach increases in accuracy due to it being in the thermally developed region and also because the surface temperature variation with χ is not as severe as at the lower values of χ. For slug flow, Sparrow and DeFarias' results in ref. [5] show that the local Nusselt number approaches a constant value as χ gets larger and, thus, one expects qualitatively similar results for the actual quadratic velocity profile, namely that the standard quasi-steady solution's accuracy increases with increasing χ. This is also to be expected, for small enough $a = \rho c_p R/\rho_w c_{pw} b$, from the analysis

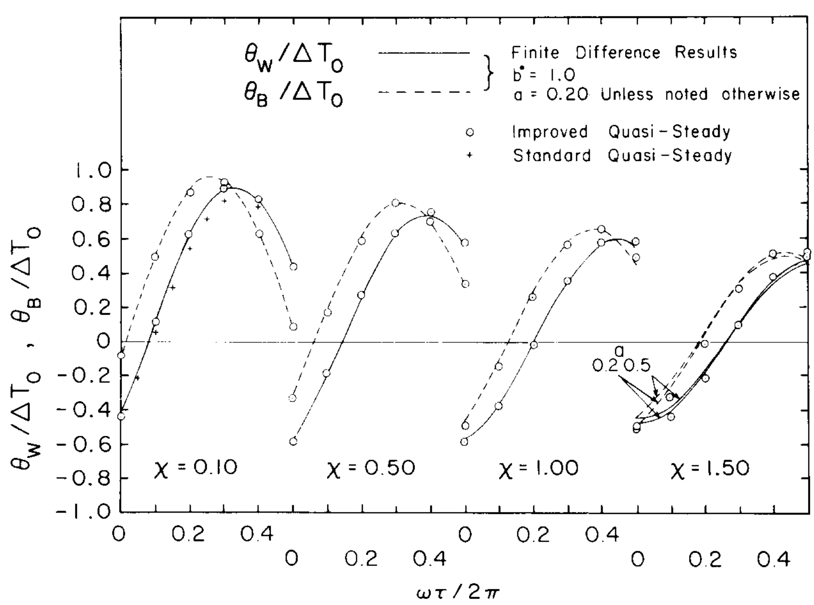

FIG. 2. Wall and bulk mean temperature excess distribution.

leading to conditions for the validity of standard quasi-steady analyses presented in Sucec [6].

Although the standard, or usual, quasi-steady procedure employs a constant value of the surface coefficient, an x dependent thermal entry length coefficient could, with increased effort, be used as was done in ref. [5]. By restricting the analysis to low values of χ, the author derived some representative results with the χ dependent Nusselt number. It was found that this type of standard quasi-steady solution yielded only modest improvement over the constant Nusselt number solution at $\chi = 0.10$, namely a few percent improvement, and considerably more improvement at the very low $\chi = 0.02$ value. But, even here, there is still a significant difference between the standard quasi-steady result and the improved quasi-steady method being advanced herein which essentially coincides with the finite-difference result. Hence, as a result, only the simpler quasi-steady results using a constant Nusselt number are given in the figures.

All of the finite-difference solutions in Figs. 1 and 2 are for a value of $a = 0.20$ except for the curves appropriate to $\chi = 1.50$ in Fig. 2 where curves for values of $a = 0.20$ and 0.50 are displayed. Clearly, the improved quasi-steady results are closer to the lower value, $a = 0.20$, as would be expected from the improved quasi-steady solutions presented in ref. [8]. From a study of Figs. 1 and 2 as well as additional results not presented herein, it appears as if the important parameter governing the accuracy of the improved quasi-steady approach in this problem is not simply a, but, rather, the product $ab^*\chi$. Tentatively, it is suggested that acceptable accuracy of the method will ensue for $ab^*\chi \leqslant 0.25$. On this basis, the agreement between the baseline finite-difference solution and the improved quasi-steady approach is deemed acceptable for all results presented in Figs. 1 and 2 with the exception of the $\chi = 1.50$ cases for both values of a. Hence, the application of, and suitability of the method has been established for the case of the actual quadratic velocity profile for flow within and beyond the thermal entrance region of a duct and these results complement some of the improved quasi-steady solutions of ref. [8] which were for the simpler case of a linear velocity profile with consideration restricted to the thermal entrance region only. This method, when it can be used with acceptable accuracy, obviates the need for the long computer runs needed in the finite-difference solution of equations (1)–(3) since some of these computer runs required 4000 s or 70 CRU.

CONCLUDING REMARKS

By means of a suitable approximation in the Laplace transform plane, a solution by the improved quasi-steady method is found for the transient, conjugated heat transfer problem of fully developed, hydrodynamic flow in a parallel plate duct with a sinusoidal inlet temperature variation. Although the solution is, nominally, for large downstream distance χ, the use of three terms, $j = 2$, allows it to be used as close to the duct entrance as $\chi = 0.02$. It is shown that, at downstream distances larger than $\chi = 0.20$, two terms in the approximation or even a simple one-term, $j = 0$, result may suffice. Comparison of the improved quasi-steady predictions of wall temperature and fluid bulk mean temperature with those of the finite-difference solution indicate close agreement between the two for values of $ab^*\chi \leqslant 0.25$ whereas the usual, standard quasi-steady approach can lead to a significant error at lower values of χ.

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CONVECTION THERMIQUE FORCEE VARIABLE CONJUGUEE DANS UN CANAL A PLANS PARALLELES

Résumé—On considère le problème instationnaire, conjugué d'un canal à plans parallèles avec un fluide dont la température à l'entrée varie périodiquement dans le temps. Des approches ont utilisé soit l'approche classique en quasi permanent, soit un profil de vitesse d'écoulement piston. Ici, le profil de vitesse quadratique, caractéristique d'un écoulement en canal établi, est utilisé dans une approche en quasi permanent précédemment présentée par l'auteur, laquelle prend en compte de façon approchée les effets de l'histoire thermique et de la capacité thermique finie du fluide. Des fonctions de réponse pour la température de paroi du canal et la température moyenne du fluide sont développées et sont comparées à une solution par différences finies et à la solution classique simple.

INSTATIONÄRE KOMBINIERTE WÄRMEÜBERTRAGUNG BEI ERZWUNGENER KONVEKTION IN EINEM KANAL AUS PARALLELEN PLATTEN

Zusammenfassung—Diese Arbeit behandelt das instationäre kombinierte Wärmeübertragungsproblem in einem Kanal aus parallelen Platten mit zeitlich periodisch schwankender Eintrittstemperatur der Flüssigkeit. Bei früheren Berechnungen wurde entweder die übliche quasistationäre Näherung oder das Geschwindigkeitsprofil einer Pfropfenströmung verwendet. Hier wird das tatsächliche parabolische Geschwindigkeitsprofil, welches für voll ausgebildete Kanalströmung charakteristisch ist, in einem verbesserten quasistationären Ansatz verwendet, der schon früher vom Autor vorgeschlagen wurde. Dieser Ansatz berücksichtigt näherungsweise die thermische Vorgeschichte und die endliche Wärmekapazität des Fluids. Es werden Reaktionsfunktionen für die Temperatur der Kanalwand und die mittlere Fluidtemperatur unter Verwendung dieses Ansatzes bestimmt und mit der Lösung eines finiten Differenzenverfahrens und des gewöhnlichen einfachen quasistationären Ansatzes verglichen.

НЕСТАЦИОНАРНЫЙ СОПРЯЖЕННЫЙ ТЕПЛОПЕРЕНОС ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ В ПЛОСКОПАРАЛЛЕЛЬНОМ КАНАЛЕ

Аннотация—Рассматривается сопряженная задача нестационарного теплопереноса в плоскопараллельном канале, когда температура жидкости на входе периодически меняется во времени. В ранее проводившихся исследованиях применялся или обычный стандартный квазистационарный метод или использовался профиль скорости ползучего течения. В данной работе реальный квадратичный профиль скорости, характерный для полностью развитого течения в канале, используется в усовершенствованном квазистационарном методе, ранее предложенном автором, который приближенно учитывает как эффекты тепловой предистории, так и конечной теплоемкости жидкости. С помощью предложенного метода выведены функции отклика для температуры стенок канала и средней по объему температуры жидкости, и проведено сравнение с решением методом конечных разностей, а также обычным квазистационарным методом.